Stan Rifkin/ELP5 - Educ 306
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Comparing patterns: elevation, scatter, and shape

Introduction for the reader: This paper was submitted for a course. It is a bit mathematical, though one can easily skip the technical bits. The course was part of the Executive Leadership Program (ELP) in the Graduate School of Education at the George Washington University in Washington DC. That program is for doctoral students, who are arranged in groups (cohorts) and take all of their courses together.

ABSTRACT. Comparing a set of measures, such as a profile, between one subject and another is not a topic dealt with in traditional statistics. Such a comparison gives a measure of the pattern match between two subjects. There is a relatively long history of the development and evaluation of such profile similarity measures, which is sketched in the paper. A few of the measures are applied to an Executive Leadership Program cohort using the Competing Values Theory of Leadership.

> "... [H]ad we to name one key weakness in the analytical approach to research, and probably in the social sciences in general, it would be that researchers have been bent upon testing for simple, circumscribed relationships instead of searching for or constructing a multiplicity of rich, revealing patterns." (Miller & Friesen, 1984, p. 18)[1]

"A pattern [profile][2] in its most general form, is a system of measurable parts related in a whole. The persistent relations which tie the parts into a whole and distinguish it from other wholes, both in fact and in human perception, range from simple qualitative and spatial relations to the most complex functional, causal, evidential, and psychological relations." (Cattell, 1949, p. 279)

Profiles are interesting because they represent a "total configuration rather than mere levels in specific variables." (Cattell, 1949, p. 279) Profiles lead to the use of "types" and wholes rather than pieces and parts. Profiles, when kept intact and not analyzed piecemeal, support synthesis, Gestalt, holism. In fact, certain research designs depend upon simultaneously interpreting many variates, of comparing patterns: it is the center of the research hypothesis (e.g., Cattell (1949) compares culture patterns of two nations).

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[1] Quick reminder to the reader that text inside square brackets in quotations is not in the original, but rather added for clarity by me.
[2] Pattern similarity and profile similarity are used interchangeably.

Perhaps the most famous profile comparison is the one presented in Fig. 1, below. It was offered in *United States v. Gotti, et al.* (1987) by Gotti's defense team. Tufte (1990, p. 31) It is a profile of the criminal activities of those persons who testified against Gotti and who were granted leniency or immunity from prosecution therefrom. Gotti was acquitted because, according to newspaper accounts, the jurors said they believed that the profiles indicated that each informant was untrustworthy.

CRIMINAL ACTIVITY OF GOVERNMENT INFORMATS

CRIME	CARDINALE	LOFARO	MALONEY	POLISI	SENATORE	FORONJY	CURRO
MURDER	x	x					
ATTEMPTED MURDER		x	x				
HEROIN POSSESSION AND SALE	x	x		x			x
COCAINE POSSESSION AND SALE	x		x	x			
MARIJUANA POSSESSION AND SALE							x
GAMBLING BUSINESS		x		x		x	
ARMED ROBBERIES	x		x	x	x		x
LOANSHARKING		x		x			
KIDNAPPING			x	x			
EXTORTION			x	x			
ASSAULT	x		x	x			x
POSSESSION OF DANGEROUS WEAPONS	x	x	x	x	x		x
PERJURY		x				x	
COUNTERFEITING					x	x	
BANK ROBBERY			x	x			
ARMED HIJACKING				x	x		
STOLEN FINANCIAL DOCUMENTS			x	x	x		
TAX EVASION				x		x	
BURGLARIES	x	x		x	x		
BRIBERY		x		x			
THEFT AUTO MONEY OTHER			x	x	x	x	x
BAIL JUMPING AND ESCAPE			x	x			
INSURANCE FRAUDS					x	x	
FORGERIES				x	x		
PISTOL SHIPPING A PRIEST	X						
SEXUAL ASSAULT ON MINOR							x
RECKLESS ENDANGERMENT							x

Figure 1. **Profile presented by counsel for John Gotti, who was acquitted in United States v. Gotti, et al. (1987). Reproduced from Tufte (1990, p. 31)**

This paper examines how to quantitatively compare patterns expressed as profiles. It presents the history of such comparative measures, the history of evaluation and critiques of them, summarizes the best current approaches, and applies a few measures to Executive Leadership Program (ELP5) cohort data as an illustration.

Sometimes profiles are represented as a tuples (that is, a string of scores) and other times as a geometric silhouette, either resembling a histogram or a radar or spider chart. Examples are illustrated throughout the paper.

Traditional statistics can compare single pairs of numbers, for example in t-tests (for a comparison of means), analysis of variance (for a comparison of variances), and tests of significance of differences (for a comparison of correlations). Traditional statistics give no guidance on how to compare profiles, patterns, signatures, hallmarks, etc., no guidance on how to compare a group or set of descriptors. It assumes that profiles will be examined visually and (all) inferences made therefrom. This visual method is common with the MMPI, 16PF, and the Competing Values Theory of Leadership (Quinn, 1988). Recent applications of t-tests to profiles can be found in, for example, O'Reilly, Chatman and Caldwell (1991), person to organization fit; Chatman (1989), person to organization fit; Caldwell and O'Reilly (1990), person to job fit; and Butler et al. (1982), personality profiles between cancer patients and other disease groups; Knights (1973/1979), children with minimal brain dysfunction; Curfs et al. (1995), children with Prader-Willi syndrome compared to others attending regular schools; Miller and Paniak (1995), MMPI and MMPI-2 profile of brain-injured individuals; Munley et al. (1995), post-traumatic stress disorder and the MMPI-2; and Ware et al. (1995), comparison of element and summary measures of the SF-36 physical and mental health survey.

One challenge of using profiles to characterize a constellation of behavior or traits (or anything else) is how to compare them, as they are a <u>set</u> of numbers, not just a single one.

Profile comparisons could investigate questions such as:

" 1. How similar are Persons 1 and 2?

" 2. How similar is Person 1 to Group Y?

" 3. How homogeneous are the members of Group Y?

" 4. How similar is Group Y to Group Z?

" 5. How much more homogeneous is Group Y than Group Z? Than the combined sample?" (Cronbach & Gleser, 1953, p. 457)

Profile comparison could be made quantitative by computing an index of similarity or congruence (as between the person and the organization, supervisor and subordinate, organizational strategy and environment, two organizations that are seeking to adopt the same technology, and person and job) (Edwards, 1993, p. 641) and using it for inferences. Cronbach and Gleser (1953, p. 457) state that profile similarity measures are descriptive and that inference related to multivariate analysis is a solved problem if the variables are normally distributed.

This paper presents, evaluates, summarizes, and applies several computed indices of similarity and uses algebra, statistics, and visual comparison to indicate some of the issues of some of the approaches to such indices. A third approach to profile characterization (the first is visual, the second an index) is cluster analysis. Cluster analysis of profile data is beyond the scope of this paper.[3]

INTRODUCTION

Profile similarity measures are usually stated in terms of elevation, scatter, and shape. To explain and illustrate these terms, assume a profile consisting of scores of k variates over N subjects, where x_{ij} is the score of person i on variable j. Elevation is the mean of all scores for a given person. Scatter is the square root of the sum of squares of the individual's deviation about his own mean (that is, the standard deviation multiplied by the square root of *k)*.[4] Shape is the residual information in the score set after controlling profiles for both elevation and scatter (intuitively shape is the actual pattern of "ups" and "downs." (Skinner, *1978, p.* 297)).

Cronbach and Closer (1953, p. 460) illustrate the first two of these quantities by supposing there are five traits *(a, b, c, d, e)* and three subjects (A, B, C) and these scores:

	a	b	c	d	e
A	2	-2	0	3	2
B	0	-4	-2	1	0
C	3	-1	3	-1	-4

Table 1. Illustrative raw scores.

Then the elevation (that is, the mean) of A is 1, B -1 and C 0. Removing elevation (by subtracting it from the score) we have:

	a	b	c	d	e
A	1	-3	-1	2	1
B	1	-3	-1	2	1

[3] Cluster analysis, along with factor analysis, is used to discover types. The notion is that if many profiles are similar then they may be describing a type, and cluster or factor analysis may be used to isolate that type. In this paper we are content to identify as few as two similar profiles, so the issue of the type described by many similar profiles is moot.

[4] Scatter is $\sqrt{\sum x_j - \mu^2 = \sigma\sqrt{k}}$ because $\sigma = \sqrt{\frac{\sum (x_j - \mu)^2}{k}}$. (Lemineur, 1971, p.2)

C	3	-1	3	-1	-4

Table 2. **Illustrative raw scores with elevation removed.**

If we remove scatter by dividing each subject's score by the scatter (4,4, and 6, respectively), then we obtain:

	a	b	c	d	e
A	1/4	-3/4	-1/4	1/2	1/4
B	1/4	-3/4	-1/4	1/2	1/4
C	1/2	-1/6	1/2	-1/6	-2/3

Table 3. **Illustrative raw scores with elevation and scatter removed.**

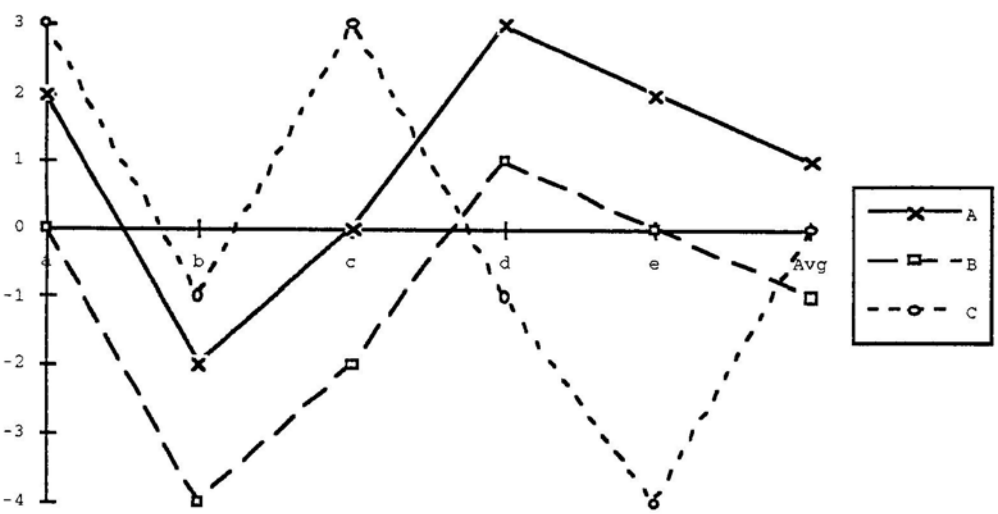

Figure 2. **Profile of illustrative raw scores.**

The values of the product-moment correlation and the rank-order correlation of A with B are both 1.0, independently of the removal elevation alone and elevation and scatter together (that is, correlation is independent of the addition or division, or both, by a constant). These tables, figure, and results will be useful in the sequel.

HISTORICAL BACKGROUND AND EVALUATION

According to Cattell (1949, p. 279), one the of the earliest explicit discussions for determining pattern similarity appeared in Zubin's (1936) analysis of patterns in questionnaire responses. Zubin suggested as a measure of pattern similarity the proportion of identical scores, $1+\frac{1}{k}\sum_{v=1}^{k}\sum_{i=1}^{N}\frac{x_v-\bar{x}_v}{\bar{x}_v-x_v}$. Cattell also mentions that Burt (1937) and Stephenson (1936) both developed the notion of "degree of similarity in personality pattern" in their Q-technique. This section continues by describing in approximate chronological order the development and evaluation of the major indices of profile similarity.

Cattell's r_p

Cattell (1949)[5] developed his r_p index in several important steps:

1. Definitions. There are fundaments and relations. Fundaments are the "atoms" of measured attribution. "For example, if height and girth are two 'dimensions' of the physical man, a small and very muscular man may possess the same ratio as a large fat man, though the 'fundaments' of this relation are very different." (p. 281) "There is ... a duality about every pattern that requires its definition be given at two levels. Equal fundaments imply equal relations but equal relations do not imply equal fundaments.... There is a hierarchy of internal relationships among fundaments and there are the fundaments themselves." (p. 282)

2. Three senses of pattern matching.

a. Shape. There are two broad ways of expressing shape: correlation coefficient and the use of ratios among the various parts. The second was not explored further because it increases linearly as the number of variates increase. Cattell asserts (p. 283) that it is "fallacious ... to say that two individuals belong to the same 'type' because they have similar shapes, when their levels are decidedly different."[6] One can make the same observation as Cattell by referring to Fig. 2, above, and asking whether subject A is the same type as B, even though they correlate perfectly.

McCrae (1993, p. 27) illustrates Cattell's point perhaps the best of the authors and critics in his illustration, Fig. 3, of the NEO five personality factors (the so-called "Big Five" in psychology, which McCrae pioneered). Each of the three profiles is a self-report in a solid line and a spouse-report of the same person in a dotted line. McCrae notes that in the top profile the shapes are identical, they are perfectly correlated, but do they describe the same type? Note that the self-report is in the high and very high ranges, and that the spouse-report is in the low and very low ranges. McCrae, in fact, asks if there is any conceivable possibility they are describing the same person. In the bottom profile there is very poor correlation, but it is conceivable that the same person,

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[5] Cattell is perhaps best known for the 16 personality factor test, a staple of personality inventories.

the same type, is being described because nearly all of the variation is in the normal range. These observations are considered in greater detail below in Index of Profile Agreement.

Cattell proposes a shape correlation coefficient as an index of pattern similarity (p. 283) as follows:

$r_s = \frac{\sum x'y'}{n\sigma_{x'}\sigma_{y'}}$, where the primed variables have been standardized with respect to the distribution within each separate category, element, or dimension of the profile. It may be helpful to recall that the product-moment correlation is $r = \frac{\sum xy}{n\sigma_x\sigma_y}$, where the variables here are in raw form, not standardized.

b. Absolute agreement. Cattell proposes another index when shape alone (that is, elevation has been abstracted) is not indicative. He states "Chi-square may suggest itself but is actually not suitable because it gives only a measure of departure of agreement from chance, whereas we seek a statistic that is quantitatively more similar to the correlation coefficient." (pp. 284-5)

c. Effect. One of the purposes of matching patterns is to confer a functional equivalence to two similar patterns. Cattell states "that a man is selected as a teacher whose profile most closely approaches the profile of other successful teachers. It is assumed that then he too will probably make a good teacher." (p. 285) Accordingly, Cattell suggests ordinary multiple linear regression as the appropriate first candidate for an index of effect similarity. The regression equation would predict the performance effect of an individual in a situation with a set of source traits (factors) that were weighted (factor-loaded).

Matching patterns for effect introduces several concepts not explicitly in correlation: interaction if the factors are not orthogonal, and direction of the difference (that is, there is no optimum level of each factor). This second observation yields an interesting and important recognition: if the relationship are indeed linear, then there is no optimum level of contribution of an individual element, any increase in the element will produce an increase in performance.

This last observation, Cattell asserts (p. 286), may be the undoing of multiple linear regression (though not necessarily for matching patterns for effect, a subject to be taken up by Edwards, below). Psychiatrists, even

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[6] We are using the term elevation to mean level in this paper.

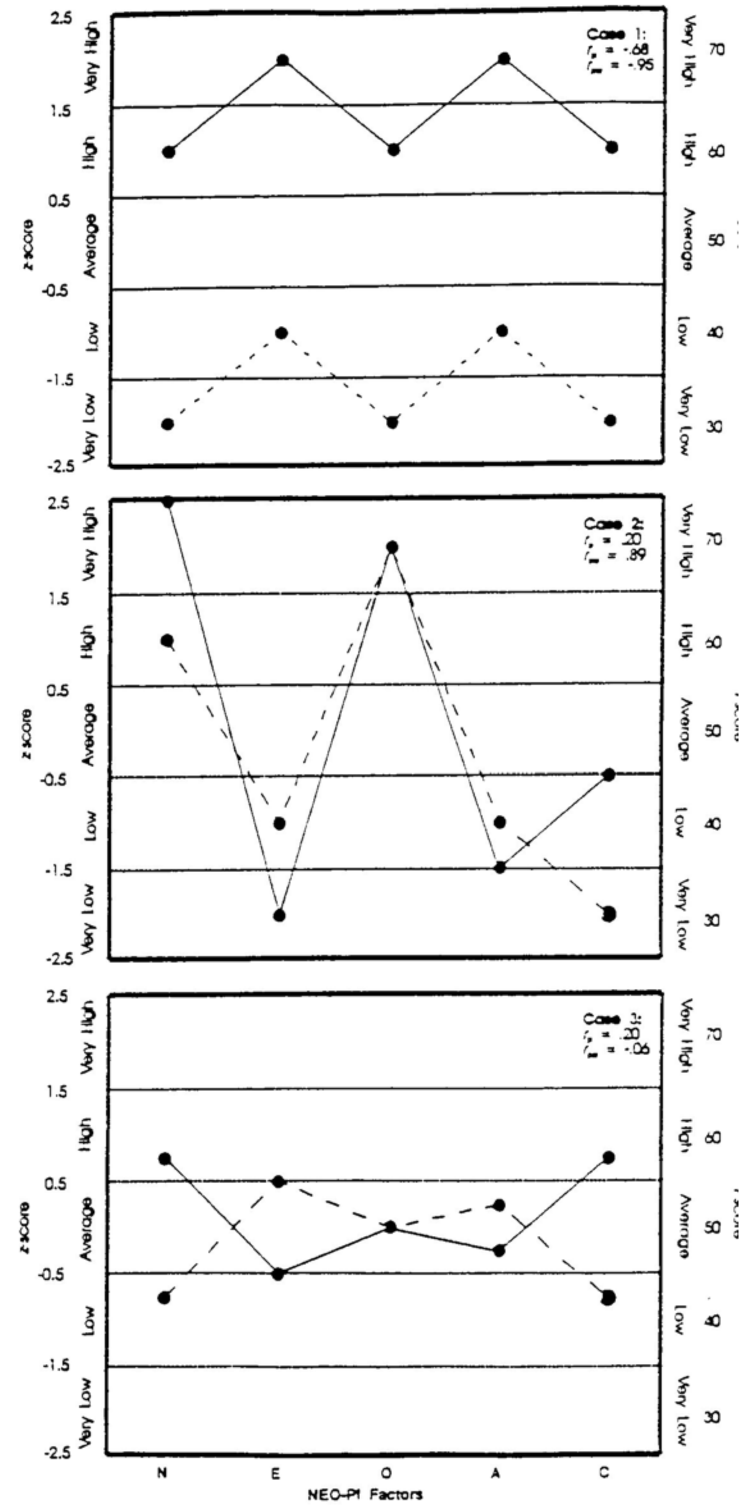

Figure 3. Hypothetical profiles for three cases on five personality factors. Self-reports are indicated with solid lines, spouse ratings with dotted lines. (McCrae, 1993, p. 27)

those who are accomplished statisticians, would not use a tool that recognized a match for a specific, narrow purpose, but rather need one that can measure effectiveness over a wide range of situations and there well may be an optimum value or optimum combination of values, so multiple linear regression is not a candidate for effect matching.

Cattell (p. 287) then defined what he came for, a new measure of pattern similarly, $r_p = \frac{2m - d^2}{2m + d^2}$ where *m* is median for χ^2 of size N,[7] and *d* is the difference in performance between two profiles (such as a criterion profile and an individual's profile). This coefficient will equal 1 when the agreement is perfect, zero when the agreement is no greater than chance, and will approach -1 when the disagreement becomes asymptotically large. A large part of Cattell's derivation of r_p, which is beyond the scope of the paper, is due to his desire to have this coefficient have the same interpretation of 1, 0, and -1 as the correlation coefficient.

Cattell argues that r_p overcomes all the objections he has levied on other candidates, namely it is interpreted the same way as the correlation coefficient, it is approximately normally distributed (i.e., not skewed), it accounts for performance or effect differences, and it is superior to chi-square alone. For all of the trouble to compute r_p, Helmstadter (1957, p. 72) reports that r_p will classify identically with the sum of squared differences, which brings us to ...

Osgood & Suci's D

One of the most common indices of profile similarity is due simultaneously to Osgood and Suci (1952); its definition was expanded by Cronbach and Gleser (1953). Using the definitions in the Introduction, Osgood and Suci (1952, p. 253) define $D = \left[\sum_{i=1}^{k} (x_i - y_i)^2 \right]^{1/2}$, where there are only two profiles, *x* and *y*, each of which is composed of *k* elements. If the *k* dimensions are mutually orthogonal, then the (Euclidean) distance D between any two points in the space of the profile is the (Pythagorean formula:) square root of the sum of the squared differences of the coordinates on the same dimension. One might ask whether the absolute differences,

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[7] The chi-square density distribution is given by Prob(*x*)= $\frac{1}{2^{\upsilon/2}\Gamma(\upsilon/2)} x^{(\nu-2)/2} e^{-x/2}$, where υ is the number of degrees of freedom and Γ is the gamma distribution. All of this to say that finding the median of the distribution is no picnic.

that is, $\sum_{i=1}^{k}\left|x_i - y_i\right|$, wouldn't serve the same function; they were suggested by Ellson (1947) (for a critique see Edwards, below).

Cronbach and Gleser (1953) investigate the space of potential coordinates by asking what happens if the raw scores at those points are transformed by removing elevation and scatter. For example, they show that D^2 = 2(1-Q), where *D* measures the distance difference of scores that have had their elevations and scatter removed, and *Q* is the product-moment correlation. (p. 461) That is, "all correlations between profiles are essentially measures of distance in *k-1* space." (p. 461) By the same token, measures where elevation has been removed are in *k-1* space and those where both elevation and scatter are removed are in *k-2* space. The significance of this is that such transformed *D*s cannot be measuring the same thing as the raw *D*, since they are the projections of the raw score *D* into fewer dimensions.

In addition, Cronbach and Gleser (1953) take Cattell's r_p to task, asking why a measure of separation should have a limit (in Cattell's case -1 to +1). "'Complete dissimilarity of persons' is an undefinable concept." (p. 462)

Cronbach and Gleser were also the first ones to caution the researcher to be careful of the research hypothesis when selecting a profile similarity index, as the inclusion or exclusion of elevation, scatter, and/or shape may influence the acceptance of the null hypothesis or a clinical diagnosis. They note that "[t]he similarity index [*D*] gives especially large weight to the first principal component among the scores or items, and therefore may be relatively insensitive to the shape or configuration of profiles." (p. 472) They also suggest the use of *D* and not D^2 because the later magnifies large distances in the squaring. (p. 471)

DuMas' coefficient of profile similarity

While the computational details obscure the heart of duMas' proposal, in essence he proposed a special case of rank-order correlation. (duMas 1946, 1947, 1949, 1950, 1953; Helmstadter, 1957). It may be of historical importance to mention that duMas also proposed a chi-square distributed measure of profile similarity, and in that sense was related to Cattell's. (duMas, 1947)

Early empirical comparisons of profile similarity

Helmstadter (1957) tested ten indices of profile similarity, most of which have already been presented above. His method was to develop 270 artificial sets of data. each having a different combination of geometrical properties (one-third spheres, one-third right circular cylinders, and one-third regular tetrahedrons). The author asserts that they represent typical counseling situations (p. 81), which I could not understand given the cryptic nature of the geometries. 180 of the sets were used to establish the parameters of each test (the inter-correlations of the variables was very low, many near zero) and 90 of the sets were used for classification. In addition, three judges also sorted the scores and they were compared with the statistical classifications from the 90 sets The results of this last comparison show that the proportion of successes ranged from a high of 0.88 to a low of 0.67; chance alone would have been 0.33. Accordingly, Helmstadter concludes that all ten measures classified significantly better than chance, (p. 79) In fact, of the ten, all but one (proportion of identical scores) scored above 80% chance of success; this is quite a clump: nine proportions of success between 81% and 88%. There was no analysis of whether the proportions were significantly different.

Muldoon and Ray mentioned (1958, p. 776) that even though Helmstadter's differences were small and many were statistically significant, they were of little practical use. They give no justification for these comments. Muldoon and Ray (1958, p. 776) also quote a study by Mosel and Roberts (1954) that concluded that five measures (presumably of the ten Helmstadter used) varied considerably not only because they were different techniques but also because it depended upon the standards to which the sample profile were compared.

In addition, citing Mosel and Ray, they mentioned that the measures of profile similarity did not behave as one would predict based on the logic of the computation. For example, two of the measures that inter-correlated the highest reflected one that only depends on shape and the other both shape and elevation. "All indices seem to have value in classifying but evidently they do it in different ways and often vary considerably as to results. And the way in which the clinician makes his decision is still an enigma." (Muldoon & Ray, 1958, p. 776)

Accordingly, Muldoon and Ray (1958) conducted the following experiment. They arbitrarily selected 20 temperament scale profiles from a large group of college students. One of the students was arbitrarily selected as the standard to which the rest would be compared. The profiles were drawn on quadrile paper with the 50th percentile shown, but none of the scales were titled. Comparisons were made by six common measures of similarity (of which five have been presented above) and 11 staff psychologists at a large neuropsychiatric hospital.

The statistical measures ordered the profiles from least similar to most similar. In this order the profiles were presented to the psychologists who were instructed likewise to order them. The resulting 17 rankings (six statistics and 11 psychologists) of the 19 profiles were inter-correlated using rank-order. This yielded a 17 x 17 matrix of orderings which was factor analyzed. Four factors emerged: shape, scatter, and elevation, and another that appeared to be unique to one psychologist. In terms of this factor analysis, the psychologists used predominantly shape to determine likeness. DuMas' coefficient of profile similarity was the statistical measure that most significantly approximated the clinicians' appraisals.[8]

Guertin's stratified classification

Guertin (1966) was the first of several to propose a several step method of comparing profiles. He suggested using factor analysis to analyze inter-profile correlation coefficients, and then within the similarly-shaped profiles thus obtained use D^2 to see if additional clustering is justified on the basis of elevation and/or scatter differences, (p. 156) The details of the computation are beyond the scope of this paper, but the use of a stage model that successively and systematically identifies sources of variation is going to turn out to be the best of all worlds.

Carroll and Field's evaluation

Not satisfied with the evaluations of Mosel and Roberts (1954), Muldoon and Ray (1958), and Helmstadter (1957), these two authors (Carroll & Field, 1974) generated seven data sets in which the relations among elevation, scatter, and shape are varied more systematically than predecessor studies. Eleven measures of profile similarity were compared (of which six have been described above). The measure that had the highest proportion of correct classifications were D and r_p. Mosel and Roberts (1954)
found that Cattell's r_p correlated most highly with the criterion of clinical judgment, and Muldoon and Ray (1958) found that duMas' r_{ps} was the most highly correlated by the same criterion.

Skinner's computation

[8] It should be noted that Spearman's rho was evaluated among the six measures and it is considered *the* statistic of rank-order correlation; duMas' coefficient is a special case of rank-order correlation that looks, in particular, at the just-adjacent neighbor as a measure of how close the ranks are to each other.

Skinner (1978) formalizes and operationalizes Guertin's (1966) stratification approach. Data sets are first seen as combinations of elevation, shape, and scatter. Shape is first evaluated and it divides the sample under study. Then each division is separately evaluated as to scatter. Subdivisions due to scatter are then separately evaluated with respect to elevation. Accordingly, in the end, the sample is subdivided in three stages with each subdivision being treated independently at the successive steps. Figure 4, directly from Skinner (1978), is his example of a sample of 300 subjects being first subdivided into two subgroups based on shape, one of which is subdivided into two additional subgroups based on scatter, each of which are again subdivided based on elevation.

Based on theory, Skinner (1978, pp. 303 ff) gives the computational details that Guertin (1966) lacked and links the computation to computerized statistical packages so that the stratification approach can be handily applied. Skinner concludes with an observation he attributes to Sneath and Sokal, "it is not at all clear at this point that a unique measure of similarity ... is possible or even desirable." (p. 307)

Tendency to extreme scores (TES)

Miley (1980) brought attention to a statistic that Cronbach had in a context not related to profile similarity called the tendency to extreme scores. TES is the sum of the squared deviations of each of the profile values from its mean in the population (B_{ij}^2 as defined in the next paragraph).

In order to follow Miley's comments on TES it is necessary to revisit Cattell's $r_p = \frac{2m - d^2}{2m + d^2}$, where m is median for χ^2 of size N, and d is the difference in performance between two profiles. Here

$$d^2 = \sum (x_{ij} - x_{gj})^2 = \sum (x_{ij} - \mu - x_{gj} + \mu)^2 = \sum (B_{ij} - D_{gj})^2 = \sum B_{ij}^{\ 2} - 2\sum B_{ij} D_{gj} + \sum D_{gj}$$

where a profile is a vector of k elements x_{ij}, where each x is the score of subject i on variable j, x_{gj} is the mean on variable j as achieved by members of group g. B_{ij} is x_{ij}-μ, and D_{gj} is x_{gj}-μ. (pp. 56-58) Later Cattell modified the formula to

$$r_p = \frac{2m + \sum D_{gj}^{\ 2} - d^2}{2m + \sum D_{gj}^{\ 2} + d^2}$$

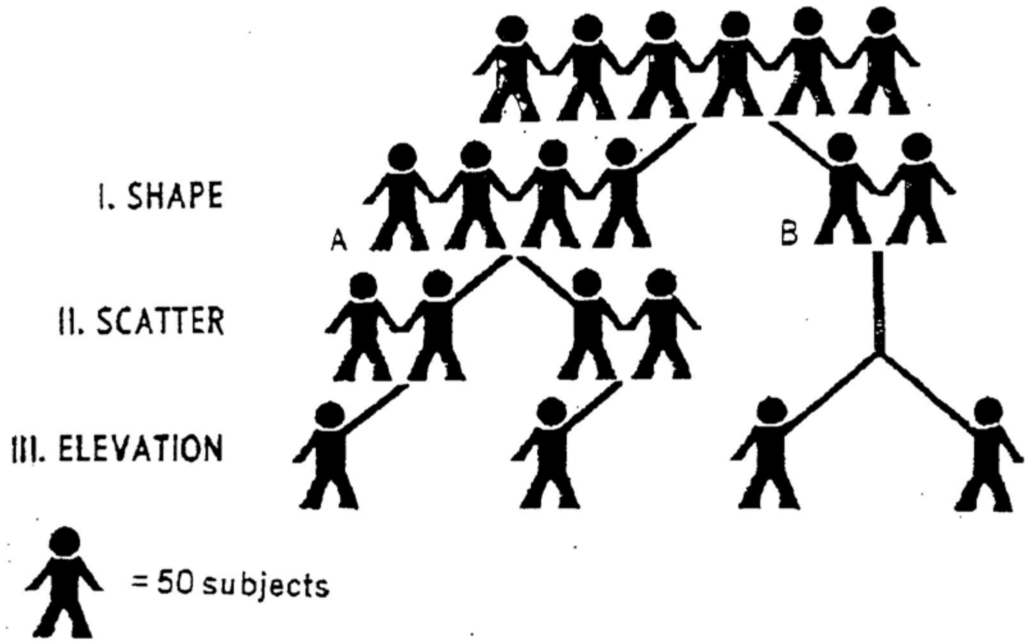

Figure 4. **Differentiating individuals according to profile shape, scatter, and elevation. (Skinner, 1978, p. 303)**

This has the effect of removing ΣD_{gj}^2 from the numerator. Miley reported that Cattell stated that the modified formula is superior to the original in that it "takes into account the extent to which a group deviates from the general population. It thus compensates for the fact that, the more deviant a group is, the more likely it is for a person to have a larger d^2 when his profile is compared with the group profile." (p. 58) Miley inferred that

> for a given subject, group deviance was seen as an artifact.... For a randomly chosen criterion group, d^2 will be small when TES is small, and hence r_p will be large. In other words, a subject with a flat or average profile will tend to be judged similar to that [criterion] group. (p. 58)

To prove his point that this is illogical, or at least users of r_p should know its limitations, Miley administered the 16PF to 590 undergraduate students and compared their scores to those of 73 occupations and 45 clinical groups in the 16 PF handbook. Thus, each subject generated 118 (73+45) values of r_p. Next, the 118 values of r_p were inter-correlated and the correlation matrix was factor analyzed. The first factor accounted for 66% of the variance. Scores on that principal factor were computed for each subject and correlated with TES (i.e., B_{ij}^2). The resulting value was -0.952, clearly identifying the factor as TES. A fortiori, $(-0.952)^2$ x 66% = approximately 60% of the r_p variance is attributable to TES.

To further appreciate this effect, Miley selected three subjects, one with the highest TES (178), one with the lowest (10), and one as close as possible to the expected value of 61.36 (62). Median values of r_p were, respectively, -0.400, 0.539, and -0.008, with ranges of -0.557 to 0.056, 0.208 to 0.777, and -0.184 to 0.216; one notes little overlap. The S with the highest TES is significantly dissimilar to 92 of the 118 groups and similar to none, the subject with the lowest TES is significantly similar to 116 of the 118 groups and dissimilar to none, and the subject whose profile is of the expected TES is neither significantly similar nor significantly dissimilar to any group!

Miley went on to tally the number of similar and dissimilar matches for the whole sample. His results indicated the results are far from what was expected clinically, intuitively, or by using the binomial distribution. He stated, "The differences between the binomial distribution and empirically obtained distributions can be attributed to the violation of the binomial assumption that the r_ps are independent." (p. 60) "The conclusion is clear - as an index of an individual's 'belongingness' to a group, r_p is severely deficient in that it is strongly influenced by TES." (p. 60)

Butler's vector model

One might ask whether a profile might be characterized by a center of gravity or centroid. If one pictures the circular diagram of the competing values framework (see an example below), then the centroid is that point on the diagram from which we could place a string and have the diagram hang evenly parallel to the floor. It is "the middle" of the odd-shaped diagram. Centroid is the term used for a plane figure and center of mass is the term for a solid object. And there are two centroids possible with our spider diagram, one that considers it made of a wire frame and one that considers it a uniformly dense disk. The formulae are different, but both employ a vector model to compute the values. Butler, below, does not limit his discussion to circular diagrams (which, anyway, could be unwound to become histograms or silhouettes) and we can only guess that he was looking to the centroid for inspiration. (See Cronbach & Gleser, 1953, p. 471, for the relationship between the centroid and the average D^2 of an individual)

Butler (1983) suggests an extension of the geometrical measure of distance by proposing a vector model. The motivation for Butler's extension is his observations that there are two drawbacks to using D:

1. It confounds elevation, scatter, and shape by combining them into a single index.

"Nunnally noted that [D] was useful only for comparisons that treated elevation, scatter, and shape simultaneously. Lorr argued that a given [D] can 'represent a large difference between two individuals on only one dimension, or the sum of many small differences on all dimensions involved.' Cronbach and Gleser suggested using three indices resulting from sequentially removing elevation, then scatter, from [D] by expressing [D] first in raw form, then in deviation form (removing elevation), then in standardized form (removing scatter). The criterion of choice among the three indices is relevance to a specific decision. Thus the problem of confounding has been solved." (p. 748, citations omitted)

2. It assumes the unlikely property of orthogonal profile dimensions. This problem has not been solved except by extracting orthogonal factors (via factor analysis), which are often difficult to interpret. (p. 749)

Heerman disagreed with Overall's objection to using [D] for correlated scores. However, his argument focused on the fact that coordinate *axes* can be defined as mutually orthogonal even though the scores are correlated. This is true; but the orthogonal axes represent factors on which the scores are projected. One is then back to factor analysis and the problem of interpreting factors. Even if the interpretation is deemed moot (i.e., degree of similarity is the only issue), there is still the inconvenient procedure of performing the factor analysis and computing factor scores. Also, if [D] is computed from orthogonal factors, f_{1i} and f_{2i} instead of x_{1i} and x_{2i}, as in [the standard equation for [D], and [D] is used to describe profile 1 by letting the f_{2i}'s=0, how would the description be interpreted? (p. 749, emphasis in original, citations omitted)

As a consequence of these objections, Butler offers generalized indices for profile description and similarity.

The elements of a profile with [k] elements can be represented by distance vectors, or "element-vectors," X_i, in [k]-dimensional Euclidean space. The scalar magnitude of each element-vector is the score on its corresponding profile element. The relative direction of the element-vectors represent the relationships or similarities among the profile elements. These directions can be described by the matrix of inter-element correlations for a sample of [N] [k]-element profiles. The correlation matrix defines, for the sample, a multi-dimensional, generally non-orthogonal coordinate system whose axes are parallel to the element-vectors.

Every profile can be represented by a resultant distance-vector, $\vec{R} = \Sigma_i \vec{X}_i$, with

> magnitude R and direction described by angles α_{Ri} with each of the [k] element-vectors. ... Distance-vectors can also be used to describe the similarity between two profiles. Once the magnitudes and directions of the two resultant vectors $\vec{R}_a$ and $\vec{R}_b$ are computed for the two profiles, a and b, the angle β_{ab} between $\vec{R}_a$ and $\vec{R}_b$ represents the alignment of a with respect to b. The magnitude of the difference, $\left|\vec{R}_a - \vec{R}_b\right|$, is the scalar difference, D_{ab}, between the two profiles. (p. 750)

Butler gives a numerical example in two dimensions. In that example there are two profile elements (variables), 1 and 2, and two subjects (i.e., profiles), a and b. In the example, the magnitude of $\vec{R}_a$ and $\vec{R}_b$ are not equal to the scalar sums of their components. $\vec{R}_a$ is directed at 23° from axis 1 and 37° from axis 2; $\vec{R}_b$ is 46° from axis 1 and 14° from axis 2. The angle between $\vec{R}_a$ and $\vec{R}_b$ is 23°. "This personalizes the concept of alignment. Alignment is one measure of profile similarity. Vector difference is another." (p. 753, citations omitted)

The magnitude of the vector difference, D_{ab}, is 1.732 in the example, the value of (classical) D is 1.236, quite far apart. Butler claims that the magnitude of the vector difference is the correct inter-profile distance, not D. In sum, Butler argues that

> [a]lignment is a useful concept because we evaluate objects, events, situations, activities, and people from our own perspectives. It is a crucial aspect of performance appraisal. Our evaluations of others' activities are inevitably subjective. They are a function of our own self-interests in relation to the images others present and our perceptions and expectations of their roles and tasks. Performance appraisals do not reflect actual performance, $\vec{R}_b$, but rather the projection of actual performance on the evaluator's own orientation, $\vec{R}_a$. Thus, appraisal = $R_b \cos \beta_{ab}$. Appraisals are a function of alignment as well as performance. (p. 754)

Miller and Friesen's comparison

Miller and Friesen (1984, ch. 2) devote much of a chapter to comparing indices of profile similarity and relating them to methods of factor and cluster analysis. I reprint here a table from their work that summarizes their comparison. In it, "mean adjusted distance" indicates that elevation has been normalized (that is, all the scores were subtracted from the average), and "mean & variance adjusted distance" means that both elevation and scatter have been normalized, which is the usual meaning of normalization. Also, Mahalanobis measure is beyond the scope of this paper, as discussed in the limitations section below.

		INFORMATION USED[a]	ASSUMPTIONS ABOUT DATA[b]	REQUIRED DATA RELIABILITY[c]	WEIGHTING OF ELEMENTS[d]	M.V.A. SUITABILITY[e]	M.V.A. INTERPRETATION[f]
D^2	Distance	L/S	R	L	V	R	D
D'^2	Mean Adjusted Distance	L/S	R	L	V	R	D
D''^2	Mean & Variance Adjusted Distance	P	R	H	V	A	E
r_s	Product-Moment Correlation	P	R	H	V	A	E
Tau/Rho	Rank Correlation	P	O	L	V	R	E
r_p	Coefficient of Pattern Similarity	L/S	R	L	V	R	D
D_m^2	Mahalanobis Measure	L/S	R	H	C	A	D
Nominal Association Measures	This category is quite variable; see Sokal and Sneath (1963)	L/S	N	L	V	R	D

a P means pattern information only, L/S indicates that level and/or scatter is also reflected in measure.
b R limits use to ratio scale or interval data, 0—ordinal, N—nominal
c L indicates data need not have very high reliability, H indicates the need for high reliability.
d V indicates that each profile element or variable is weighted equally (unless researcher explicitly decides otherwise), C indicates that each principle component or dimension is weighted equally.
e A indicates measure is suitable for all or most forms of multivariate analysis (MVA), R means that use is restricted (usually to exclude factor analysis).
f E means interpretation of MVA is easy, 0 indicates there might be some difficulty in determining why a particular grouping occurred.
Source: Danny Miller, The Role of Multivariate Q-Techniques in the Study of Organizations, *Academy of Management Review* 3 (July 1978).

Figure 5. **Comparison of profile similarity measures. (Miller & Friesen, 1984, p. 42)**

Edwards' emphasis on outcomes

Recall that Cattell (1943, p. 285) first remarked that one aspect of profile similarity could be its ability to predict outcomes, or the same quantity that is predicted by a regression equation. Edwards (1993, 1994) expands Cattell's conjecture appreciably by taking in turn seven measures of profile similarity and showing how each, when seen as a regression equation, constrains the solution. I shall use a simple illustration as indicative of Edwards' insight.

Assume that the algebraic difference between two components, X and Y, is being used as an index of profile similarity. The regression equation of the index, Z, would be $Z=b_0+b_1(X-Y)+e=b_0+b_1X-b_1Y+e$, where e is the regression error term. However, normally, the regression equation of two independent variables, X and Y, would be $Z = b_0+b_1X+b_2Y+e$. Therefore, for the two equations to be the same, it must be the case that b_1 of the first equation equals $-b_2$ of the second. The question Edwards raises is whether this constraint was intended by the user of the index of profile similarity.

When one examines, as Edwards does (1994, pp. 54-55), each of seven popular measures of profile similarity by being substituted into a regression equation, then the results are quite startling in that most of the measures imply numerous constraints, virtually none of which are ever tested in empirical evaluations.

Accordingly, Edwards proposes an alternative general procedure based three propositions (1994, p. 72):

1. The relationship between profile similarity (Edwards uses the term congruence) and an outcome should be considered in three dimensions: the paired components each on one axis and the outcome on the third. This keeps the components orthogonal with each other and with the outcome.
2. The relationship between the profile and the outcome should be viewed as a three-dimensional surface. After all, that is the way to view a three-dimensional relationship.
3. The constraints of any index of profile similarity should not be imposed on the data, but rather should be viewed as hypotheses to be empirically tested. If the tests confirm the constraints, then the index can be taken as empirically validated.

The alternative approach assumes that the component measures are commensurate, at least at the interval level, and share the same scale. These assumptions assure the conceptual relevance of the component measures to one another and are necessary to meaningfully interpret profile similarity, and the interval measurement is required for the regression analysis.

Edwards' alternative approach contains these steps (1994, p. 73):

1. One or more indices of profile similarity is selected and the corresponding regression equation generated.

2. Each index is computed and then one establishes that:

   a. The proportion of variance explained by the regression equation is significant,

   b. Appropriate regression coefficients are significant and in the right direction,

   c. The implied constraints are valid, and

   d. No higher-order terms beyond those indicated by the regression model are significant.[9]

As one can observe from the steps above, Edwards' proposed general approach is phased or tiered, though in a different way than Skinner (1978).

Coefficient of Profile Agreement

McCrae (1993) proposed an adjustment to Cattell's coefficient of pattern similarity, r_p, for cross-observer agreement; that is, a coefficient of agreement between two or more observers of the same phenomena or person. The need for an unproved coefficient is indicated in Figure 3, above. In Case 2, for example, $r_p = 0.20$ and the proposed $r_{pa} = 0.89$; that is, there is a substantial disagreement about the similarity. McCrae argues (1993, p. 26) that it appears that both profiles are describing the same person, else how could one account for agreement where the self scores are, by and large, so extreme. Therefore, the coefficient of agreement in Case 2 should be high, unlike the value of r_p. Case 3 is interesting because the distances between self-report and spouse report are exactly the same as in Case 2, but the level has been centered around the middle of the scale. Accordingly, r_p is the same, 0.20. But McCrae reasons (1993, pp. 26-27) that in Case 3 there is substantially less agreement precisely because the scores are so, well, average. In other words, McCrae would like to give more weight to agreement in extreme scores because they are much less likely to differ due to chance. This, of course, is in harmony with Miley's observations, above.

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[9] Higher-order terms are generated when the regression equation contains quantities that must be multiplied and so polynomials can arise in orders higher than the original factors.

Accordingly, McCrae proposes (1993, p. 29) $r_{pa} = \dfrac{I_{pa}}{\sqrt{(m-2)+I_{pa}{}^2}}$, where

$I_{pa} = \dfrac{m + 2\sum M^2 - \sum d^2}{\sqrt{10m}}$, where a profile of *m* elements is considered, the sum will have a mean of *-m* and a variance of *10m, M* is the mean of two ratings for each profile element, and *d,* as in r_p, is the difference between standardized ratings. McCrae called I_{pa} the index of profile agreement and r_{pa} the coefficient of profile agreement. McCrae claims that r_{pa} has approximately the same statistical significance of the Pearson correlation coefficient based on the same number of cases.

McCrae conducted an empirical study to compare r_{pa} with other indices of profile similarity, in this case raw distance, Euclidean distance (Overall, 1964), and vector distance (Butler, 1983). r_{pa} was consistent across the empirical test and tended to be intermediate in value among the other distance measures.

LIMITATIONS OF THE LITERATURE REVIEW

The following profile similarity measures have been mentioned in the original articles cited above. In a future work they might be presented in greater detail:

1. Mahalanobis (1936). In particular, Overall (1964) uses this measure as the basis of dealing with the joint issues originally raised by Cronbach and Gleser (1953) that *D* is only applicable if (a) the distances are on the same scale (which is difficult when dealing with affect), and (b) the axes along with the distances are being measured are uncorrelated. Cronbach and Gleser observed that the D statistic computed over correlated profile elements is equivalent to a D statistic computed from the underlying orthogonal factor variates, but each weighted according to the proportion of total variation accounted for by that factor. (Overall, 1964, p. 196) Overall proposes $D^2 = d^T C^{-1} d$, where *d* is the vector of difference scores, possibly along correlated axes, d^T is the transpose of *d,* and C^{-1} is the inverse of the variance-covariance matrix. This formulation accounts for the possible intercorrelation of the factors. He then deduces two measures that will be detailed later in history by Butler (1983): the direction and distance from the origin of factors that have been translated into vectors.
2. duMas (1946, 1947, 1949, 1950, 1953).
3. Nunnally (1962), whose comments prompted Overall (1964) and Butler (1983) to respond.
4. Rank order.

5. Cohen's $R_{c'}$, an index of profile similarity that is invariant over variable reflections or changes in the direction of measurement. (Cohen, 1969; Paunonen, 1984)

6. Linear discriminant.

7. Q-sort. The most current seminal work is Funder et al. (1993).

8. Cluster and factor analysis. For those readers competent in French, Lemineur (1971) provides worked examples comparing values of D and r_p with those obtained by cluster and factor analysis.

9. Numerical taxonomy, systematics, and typologies. (e.g., Sokal & Sneath, 1963; Miller & Friesen, 1984, ch. 2)

APPLICATION TO COHORT DATA

Quinn (1988) proposed a framework for explaining excellent managers and leaders, the competing values theory of leadership. In a word, Quinn found that there existed eight management roles and that the best managers rated near the maximum possible scores for all eight roles. And that the best managers performed in the roles in seratum, as it was appropriate to. Accordingly, it would miss Quinn's point to use, for example, the average score to characterize a manager, rather it would be appropriate to use a measure that captured the overall configuration of the pattern of scores.

The following table indicates some general "demographics" of the 18 subjects in the cohort with respect to the competing value framework.

Competing values role	**Average**	**Standard deviation**
Innovator	6.25	0.58
Broker	6.01	0.65
Producer	6.28	0.80
Director	4.50	1.46
Coordinator	3.58	1.63
Monitor	3.17	1.31
Facilitator	6.06	0.68
Mentor	5.78	0.75
Overall	5.20	1.59

As a measure of profile similarity, I have computed Osgood and Suci's D. The results are a lower triangular matrix, below.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	3.0																
3	3.9	4.4															
4	3.0	2.6	3.5														
5	3.5	2.6	4.1	3.5													
6	2.8	5.1	4.3	5.1	4.9												
7	3.6	3.7	4.1	2.8	3.4	5.0											
8	3.3	3.3	6.3	4.5	4.4	4.9	5.6										
9	2.9	4.1	3.9	3.6	3.6	3.4	2.1	5.2									
10	5.4	4.0	4.6	4.1	3.6	6.2	3.0	6.5	4.0								
11	3.7	2.8	3.9	2.8	2.6	5.2	1.6	5.1	2.8	2.3							
12	3.5	5.3	5.4	5.7	5.5	3.7	6.8	4.7	5.7	8.1	6.8						
13	1.8	3.7	4.2	4.3	3.8	2.3	4.8	3.5	3.5	6.0	4.7	2.6					
14	2.8	4.5	5.7	5.1	4.7	3.3	5.7	3.1	4.4	7.1	5.8	2.8	1.7				
15	2.9	3.2	5.0	3.4	4.2	5.2	4.8	3.5	4.5	6.3	4.6	4.3	3.3	3.5			
16	2.6	4.0	3.8	4.2	3.9	3.4	5.4	4.0	4.5	6.5	5.2	2.2	1.9	2.6	3.1		
17	4.6	5.0	2.2	4.3	4.5	5.2	5.3	6.8	5.4	5.8	5.1	5.0	4.9	6.3	5.5	4.0	
18	2.4	4.1	4.7	3.9	3.9	3.8	4.2	4.5	3.2	6.0	4.6	3.7	2.4	2.7	3.0	3.1	5.2

Without trying to be an "eye chart," the matrix contains two values of interest: the greatest distance, 8.1, and the smallest distance, 1.6. The pairs of subjects with those distinguishing profiles are illustrated below.

While it is difficult to infer anything from the most different pair, except, perhaps, that they are indeed dissimilar, it is possible to infer from the two most similar profiles that not only are they similar in shape, they are similar in their extreme values (viz., the three scores around 2.0). Therefore, it was not necessary to resort to any of the more sophisticated measures or procedures to indicate profile similarity. This result is not generalizable!

Also, these empirical results, as crude as they are, do illustrate what was intended by the paper in the first place: the table of averages and standard deviations, so often found in traditional papers, does not indicate anything about pattern similarity.

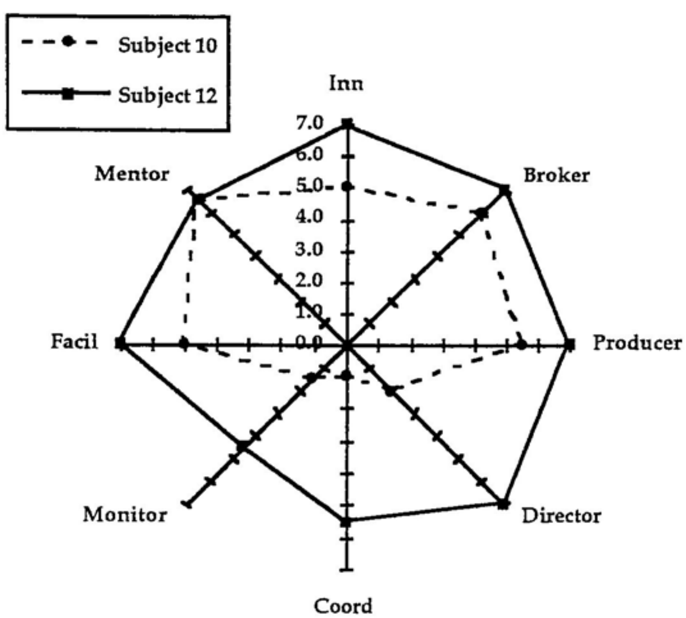

Figure 6. **Comparison of the two most different ELP cohort members based on responses to the competing values framework questionnaire.**

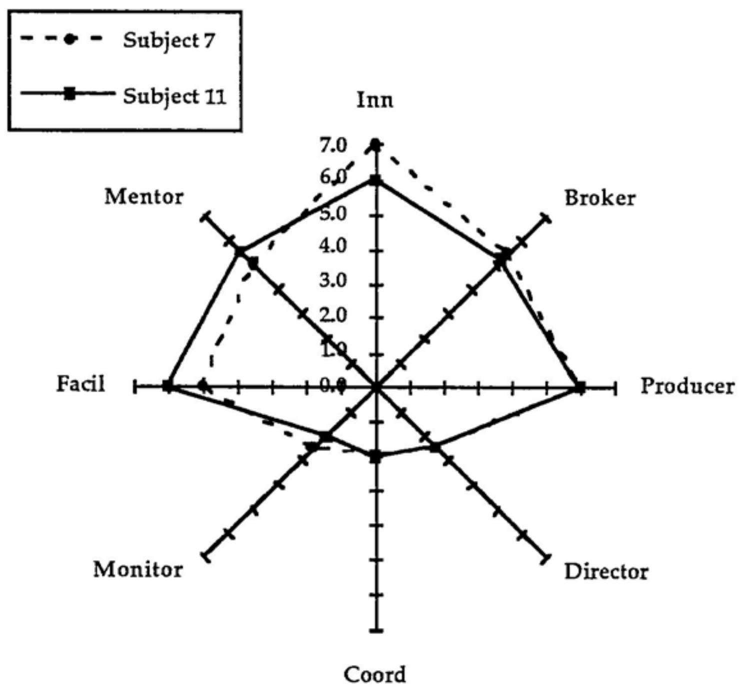

Figure 7. **Comparison of the two most similar ELP cohort members based on responses to the competing values framework questionnaire.**

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

As Skinner (1978) stated, it is imperative to identify the research goals of a particular experiment and select the profile similarity index or indices accordingly. The most conservative course would be to report elevation, scatter, and shape, not to collapse them all into a single number.

If a single index is determined to be the appropriate choice, then based on the status of current evaluations either D or r_p appear to have the best performance, despite the important theoretical limitations mentioned above (esp. Edwards, 1993 & 1994). Further research is needed to address:

- The statistical significance of the similarity measures (i.e., the probability of making a Type I error).
- The power of D and r_p, and their competitors and alternatives.
- Non-parametric alternatives, that is, statistics that do not depend upon assumptions (of orthogonality, for example).
- Application of regression to the evaluation of additional indices of profile similarity, à la Edwards, for

example, r_p.

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